An in-between "implicit" and "explicit" complexity: Automata

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> classes. By implicit, we here mean that classes are not given by constraining the amount of resources a *machine* is allowed to use, but rather by imposing linguistic constraints on the way *algorithms* are formulated. This idea has de-

(Dal Lago, 2011, p. 90)

Question

Is there an in-between?

Answer

One can also restrict the *quality* of the resources.

ICC is

machine-independent **and** without explicit bounds.

Machine-dependant

Turing machine,
Random access machine,
Counter machine, . . .

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Machine-independant

Bounded recursion on notation (Cobham, 1965),
Bounded linear logic (Girard et al., 1992),
Bounded arithmetic (Buss, 1986), . . .

Machine-dependant	**Machine-independant**
Turing machine, Random access machine, Counter machine, . . .	Bounded recursion on notation (Cobham, 1965), Bounded linear logic (Girard et al., 1992), Bounded arithmetic (Buss, 1986), . . .

The rules for storage naturally induce polynomials:

Storage $\dfrac{!_{\vec{y}}\Gamma \vdash A}{!_{x\vec{y}}\Gamma \vdash !_x A}$ *Weakening* $\dfrac{\Gamma \vdash B}{\Gamma, !_0 A \vdash B}$

Contraction $\dfrac{\Gamma, !_x A, !_y A \vdash B}{\Gamma, !_{x+y} A \vdash B}$ *Dereliction* $\dfrac{\Gamma, A \vdash B}{\Gamma, !_1 A \vdash B}$.

(Girard et al., 1992, p. 18)

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Implicit bounds		Descriptive complexity (Fagin, 1973), Recursion on notation (Bellantoni and Cook, 1992), Tiered recurrence (Leivant, 1993), . . .

	Machine-dependant	**Machine-independant**
Explicit bounds	Turing machine, Random access machine, Counter machine, . . .	Bounded recursion on notation (Cobham, 1965), Bounded linear logic (Girard et al., 1992), Bounded arithmetic (Buss, 1986), . . .
Implicit bounds	Automaton, Auxiliary pushdown machine, (Boolean circuit,) . . .	Descriptive complexity (Fagin, 1973), Recursion on notation (Bellantoni and Cook, 1992), Tiered recurrence (Leivant, 1993), . . .

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ounds	Automaton, Auxiliary pushdown machine,	Descriptive complexity (Fagin, 1973), Recursion on notation (Bellantoni and Cook,

related to the foregoing question. More specifically, we have attempted to characterize several tape and time complexity classes of Turing machines in terms of devices whose definitions involve only ways in which their infinite memory may be manipulated and no restrictions are imposed on the amount of memory that they use. The basic model

(Ibarra, 1971, p. 88)

Definition (2NFA(k, j))

For $k \geqslant 1$, $j \geqslant 0$, a 2-*way non-deterministic finite automaton with* k*-heads and* j *pushdown stacks* is a tuple $M = \{\mathbf{S}, A, B, \rhd, \lhd, \boxdot, \sigma\}$ where:

- $\mathbf{S}$ is the finite set of *states*;
- A is the *input alphabet*, B is the *stack alphabet*;
- $\rhd$ and $\lhd$ are the *left* and *right endmarkers*, $\rhd, \lhd \notin A$;
- $\boxdot$ is the *bottom symbol of the stack*, $\boxdot \notin B$;
- $\sigma \subseteq (\mathbf{S} \times (A \cup \{\rhd, \lhd\})^k \times (B \cup \{\boxdot\})^j) \times (\mathbf{S} \times \{-1, 0, +1\}^k \times \{\texttt{pop}, \texttt{peek}, \texttt{push}(b)\}^j)$

$\mathbf{2NFA(k, l)} = \{\mathcal{L}(M) \mid M \text{ a } 2\mathrm{NFA}(k, j)\}$

$\mathbf{2NFA(*, l)} = \cup_{k \geqslant 1} \mathbf{2NFA(k, j)}$

Theorem (Main characterizations)

Automata	***Language / Predicate***
$\mathbf{2NFA}(\mathbf{1},\mathbf{2})$	*Computable*
$\mathbf{2NFA}(*,\mathbf{1})$	*Polynomial time*
$\mathbf{2NFA}(*,\mathbf{0})$	*Logarithmic space*
$\mathbf{2NFA}(\mathbf{1},\mathbf{1})$	*Context-free*
$\mathbf{2NFA}(\mathbf{1},\mathbf{0})$	*Regular*

$\mathbf{2NFA}(\mathbf{1},\mathbf{2}) \supseteq$ Computable.

Let P be computable

Let M be a Turing Machine that computes it

Take M' with 1 read-only head and 1 read-write tape

Simulate the content of the tape with the pushdown tapes

□

$\mathbf{2NFA}(\mathbf{1},\mathbf{2}) \subseteq$ Computable.

Finite automata are restrictions of Turing Machines. □

2NFA$(*, \mathbf{1}) \supseteq$ **Polynomial Time.**

Restrict the Turing Machine. □

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A plain simulation? □

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> In fact, given a 2N PDA (or 2D PDA) P, it is possible to effectively design from P a 2N PDA (or 2D PDA) P', which in addition to stimulating the behavior of P scanning any input string $w = a_2 \cdots a_{n-1}$, also counts from 1 to 2^n between each move made by P. The number of

Aho, Hopcroft, and Ullman, 1968, p. 197

Your attention please

Memoization will be explained on the white board.

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Some FA with exponential runs can be simulated in linear time (Cook, 1971).

$\mathbf{2NFA}(*, \mathbf{0}) \subseteq$ Logarithmic space.

Write and update the heads' addresses in log-space. □

$\mathbf{2NFA}(*, \mathbf{0}) \supseteq$ Logarithmic space.

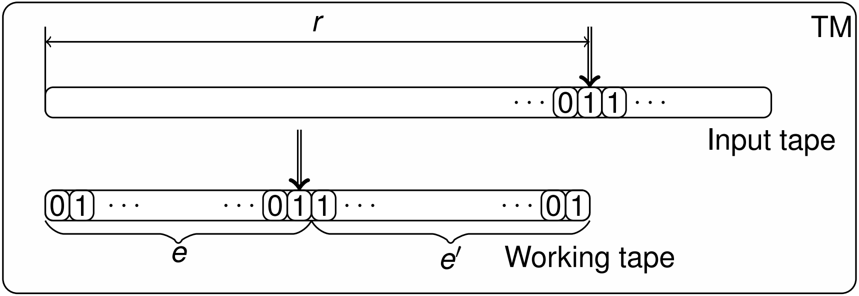
Encode the content of the tapes as positions (tallies). □

Remark (Hofmann and Schöpp, 2010)

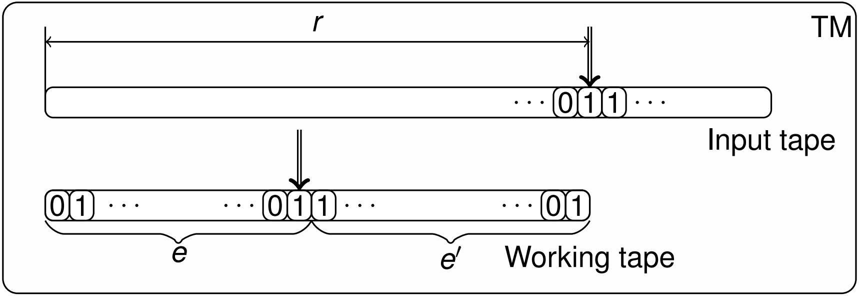
The input tape *must* be ordered.

Reminder

A binary string of length $\log(|n|)$ cannot express an integer greater than $|n|$.



TM
r
· · · 0 1 1 · · ·
Input tape
0 1 · · · · · · 0 1 1 · · · · · · 0 1
e
e′
Working tape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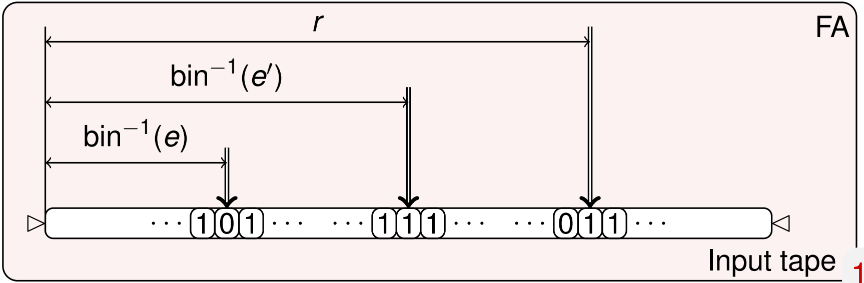
r
FA
$\mathrm{bin}^{-1}(e')$
$\mathrm{bin}^{-1}(e)$
· · · 1 0 1 · · · · · · 1 1 1 · · · · · · 0 1 1 · · ·
Input tape

$\mathbf{2NFA}(\mathbf{1}, \mathbf{1})$ = Context-free
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Theorem (Equalities)

$$\mathbf{2NFA}(*,\mathbf{0}) = \mathbf{NL}$$

$$\mathbf{2DFA}(*,\mathbf{0}) = \mathbf{L}$$

$$\mathbf{2NFA}(*,\mathbf{1}) = \mathbf{2DFA}(*,\mathbf{1}) = \mathbf{P}$$

$$\mathbf{1DFA}(\mathbf{1},\mathbf{0}) = \mathbf{2DFA}(\mathbf{1},\mathbf{0}) = \mathbf{1NFA}(\mathbf{1},\mathbf{0}) = \mathbf{2DFA}(\mathbf{1},\mathbf{0})$$

Theorem (Inequalities)

$k > 1, j \leqslant 1$

$\_\_\mathbf{FA}(\mathbf{k}, \mathbf{j}) \subsetneq \_\_\mathbf{FA}(\mathbf{k}+\mathbf{1}, \mathbf{j})$	(Too many to list)
$\mathbf{1DFA}(\mathbf{k}, \mathbf{0}) \subsetneq \mathbf{2DFA}(\mathbf{k}, \mathbf{0})$	(Holzer et al., 2008)
$\mathbf{1NFA}(\mathbf{k}, \mathbf{0}) \subsetneq \mathbf{2NFA}(\mathbf{k}, \mathbf{0})$	(")
$\mathbf{1DFA}(\mathbf{k}, \mathbf{0}) \subsetneq \mathbf{1NFA}(\mathbf{2}, \mathbf{0})$	(Yao and Rivest, 1978)
$\mathbf{1NFA}(\mathbf{k}, \mathbf{0}) \subseteq \mathbf{2DFA}(\mathbf{k}, \mathbf{0})$	(Sudborough, 1977b)
$\mathbf{2NFA}(\mathbf{k}, \mathbf{0}) \subseteq \mathbf{2NFA}(\mathbf{2} \times \mathbf{k}, \mathbf{1})$	(")
$\mathbf{2NFA}(\mathbf{k}, \mathbf{1}) \subseteq \mathbf{2DFA}(\mathbf{4} \times \mathbf{k}, \mathbf{1})$	(Sudborough, 1977a)
$\mathbf{1DFA}(\mathbf{k}, \mathbf{1}) \subsetneq \mathbf{1NFA}(\mathbf{k}, \mathbf{1})$	(Chrobak, 1986)
$\mathbf{1DFA}(\mathbf{k}, \mathbf{1}) \subsetneq \mathbf{1NFA}(\mathbf{k}, \mathbf{1})$	(")
$\mathbf{NL} = \mathbf{L}$ *iff* $\mathbf{1NFA}(\mathbf{2}, \mathbf{0}) \subseteq \mathbf{2DFA}(\mathbf{k}, \mathbf{0})$	(Sudborough, 1973)

Automata are

- well-studied[1];
- closely related to complexity;
- a place where many fundamental techniques have been discovered;
- *implicit* in an *unexpected* way;
- *explicit* in an *expected* way.

The *quality* of their storage impacts directly on their expressive power.

[1] Over-studied?

Inspiration to formalize computation in GoI (Aubert and Seiller, 2014; Aubert and Seiller, 2015).

Theorem (Aubert, Bagnol, Pistone, and Seiller, 2014)

"Logic programs where all the variables are at the same height characterize Logarithmic space."

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Thanks!

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