

An in-between “implicit” and “explicit” complexity: Automata

DICE 2015

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12 April 2015

classes. By implicit, we here mean that classes are not given by constraining the amount of resources a *machine* is allowed to use, but rather by imposing linguistic constraints on the way *algorithms* are formulated. This idea has de-

(Dal Lago, 2011, p. 90)

Question

Is there an in-between?

Answer

One can also restrict the *quality* of the resources.

ICC is

machine-independent **and** without explicit bounds.

Machine-dependant

Turing machine,
Random access machine,
Counter machine, . . .

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Random access machine,
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Machine-independant

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The rules for storage naturally induce polynomials:

$$\text{Storage} \quad \frac{!_y \Gamma \vdash A}{!_{xy} \Gamma \vdash !_x A}$$

$$\text{Weakening} \quad \frac{\Gamma \vdash B}{\Gamma, !_0 A \vdash B}$$

$$\text{Contraction} \quad \frac{\Gamma, !_x A, !_y A \vdash B}{\Gamma, !_x+y A \vdash B}$$

$$\text{Dereliction} \quad \frac{\Gamma, A \vdash B}{\Gamma, !_1 A \vdash B}$$

Explicit bounds

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Explicit bounds	Turing machine, Random access machine, Counter machine, ...	Bounded recursion on notation (Cobham, 1965), Bounded linear logic (Girard et al., 1992), Bounded arithmetic (Buss, 1986), ...
Implicit bounds		Descriptive complexity (Fagin, 1973), Recursion on notation (Bellantoni and Cook, 1992), Tiered recurrence (Leivant, 1993), ...

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Explicit bounds	Turing machine, Random access machine, Counter machine, ...	Bounded recursion on notation (Cobham, 1965), Bounded linear logic (Girard et al., 1992), Bounded arithmetic (Buss, 1986), ...
Implicit bounds	Automaton, Auxiliary pushdown machine, (Boolean circuit,) ...	Descriptive complexity (Fagin, 1973), Recursion on notation (Bellantoni and Cook, 1992), Tiered recurrence (Leivant, 1993), ...

	Machine-dependant	Machine-independant
Explicit bounds	Turing machine, Random access machine, Counter machine, ...	Bounded recursion on notation (Cobham, 1965), Bounded linear logic (Girard et al., 1992), Bounded arithmetic (Buss, 1986), ...
Bounds	Automaton, Auxiliary pushdown machine "Dooler"	Descriptive complexity (Fagin, 1973), Recursion on notation (Bellantoni and Cook, 1999)

related to the foregoing question. More specifically, we have attempted to characterize several tape and time complexity classes of Turing machines in terms of devices whose definitions involve only ways in which their infinite memory may be manipulated and no restrictions are imposed on the amount of memory that they use. The basic model

(Ibarra, 1971, p. 88)

Definition (2NFA(k, j))

For $k \geq 1, j \geq 0$, a 2-way non-deterministic finite automaton with k -heads and j pushdown stacks is a tuple

$M = \{\mathbf{S}, A, B, \triangleright, \triangleleft, \sqcup, \sigma\}$ where:

\mathbf{S} is the finite set of states;

A is the input alphabet, B is the stack alphabet;

\triangleright and \triangleleft are the left and right endmarkers, $\triangleright, \triangleleft \notin A$;

\sqcup is the bottom symbol of the stack, $\sqcup \notin B$;

$$\sigma \subseteq (\mathbf{S} \times (A \cup \{\triangleright, \triangleleft\})^k \times (B \cup \{\sqcup\})^j) \\ \times (\mathbf{S} \times \{-1, 0, +1\}^k \times \{\text{pop}, \text{peek}, \text{push}(b)\}^j)$$

$$\mathbf{2NFA}(k, j) = \{\mathcal{L}(M) \mid M \text{ a } \mathbf{2NFA}(k, j)\}$$

$$\mathbf{2NFA}(*, j) = \bigcup_{k \geq 1} \mathbf{2NFA}(k, j)$$

Theorem (Main characterizations)

Automata	Language / Predicate
2NFA(1, 2)	<i>Computable</i>
2NFA(*, 1)	<i>Polynomial time</i>
2NFA(*, 0)	<i>Logarithmic space</i>
2NFA(1, 1)	<i>Context-free</i>
2NFA(1, 0)	<i>Regular</i>

$2\text{NFA}(1, 2) \supseteq \text{Computable}.$

Let P be computable

Let M be a Turing Machine that computes it

Take M' with 1 read-only head and 1 read-write tape

Simulate the content of the tape with the pushdown tapes



$2\text{NFA}(1, 2) \subseteq \text{Computable}.$

Finite automata are restrictions of Turing Machines.



2NFA(*, 1) \supseteq Polynomial Time.

Restrict the Turing Machine.

2NFA(*, 1) \subseteq Polynomial Time.

~~A plain simulation?~~

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~~A plain simulation?~~ □

In fact, given a 2N PDA (or 2D PDA) P , it is possible to effectively design from P a 2N PDA (or 2D PDA) P' , which in addition to stimulating the behavior of P scanning any input string $w = a_2 \cdots a_{n-1}$, also counts from 1 to 2^n between each move made by P . The number of

Aho, Hopcroft, and Ullman, 1968, p. 197

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Corollary

Some FA with exponential runs can be simulated in linear time (Cook, 1971).

2NFA(*, **0**) \subseteq Logarithmic space.

Write and update the heads' addresses in log-space.

2NFA(*, **0**) \supseteq Logarithmic space.

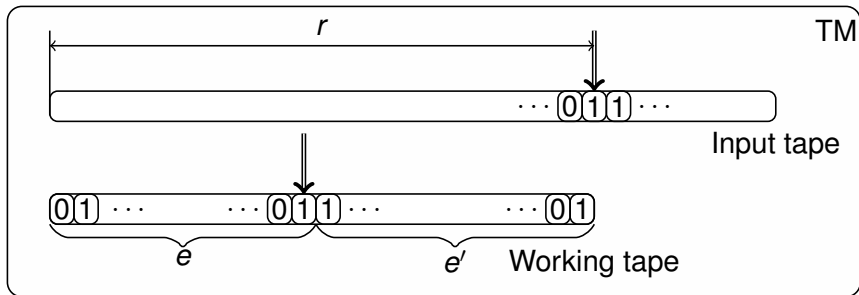
Encode the content of the tapes as positions (tallies).

Remark (Hofmann and Schöpp, 2010)

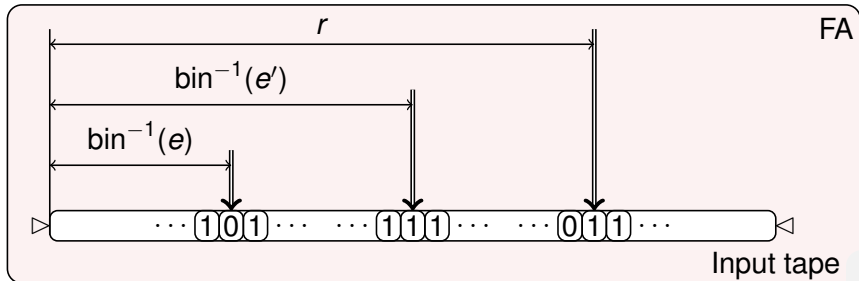
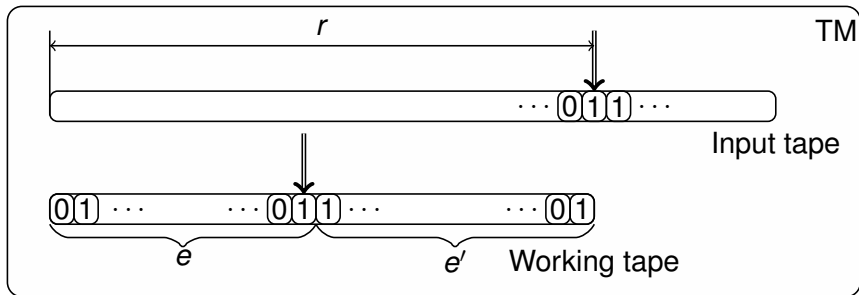
The input tape *must* be ordered.

Reminder

A binary string of length $\log(|n|)$ cannot express an integer greater than $|n|$.



Automata and Complexity: Main Characterizations



2NFA(1, 1) = Context-free

and

2NFA(1, 0) = Regular.

So well-known it became the definition. □

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Theorem (Equalities)

$$\mathbf{2NFA(*, 0) = NL}$$

$$\mathbf{2DFA(*, 0) = L}$$

$$\mathbf{2NFA(*, 1) = 2DFA(*, 1) = P}$$

$$\mathbf{1DFA(1, 0) = 2DFA(1, 0) = 1NFA(1, 0) = 2DFA(1, 0)}$$

Theorem (Inequalities)

$$k > 1, j \leq 1$$

$$\text{__FA}(k, j) \not\subseteq \text{__FA}(k + 1, j) \quad (\text{Too many to list})$$

$$1\text{DFA}(k, 0) \not\subseteq 2\text{DFA}(k, 0) \quad (\text{Holzer et al., 2008})$$

$$1\text{NFA}(k, 0) \not\subseteq 2\text{NFA}(k, 0) \quad (")$$

$$1\text{DFA}(k, 0) \not\subseteq 1\text{NFA}(2, 0) \quad (\text{Yao and Rivest, 1978})$$

$$1\text{NFA}(k, 0) \subseteq 2\text{DFA}(k, 0) \quad (\text{Sudborough, 1977b})$$

$$2\text{NFA}(k, 0) \subseteq 2\text{NFA}(2 \times k, 1) \quad (")$$

$$2\text{NFA}(k, 1) \subseteq 2\text{DFA}(4 \times k, 1) \quad (\text{Sudborough, 1977a})$$

$$1\text{DFA}(k, 1) \not\subseteq 1\text{NFA}(k, 1) \quad (\text{Chrobak, 1986})$$

$$1\text{DFA}(k, 1) \not\subseteq 1\text{NFA}(k, 1) \quad (")$$

$$\text{NL} = \text{L} \text{ iff} \quad (\text{Sudborough, 1973})$$

$$1\text{NFA}(2, 0) \subseteq 2\text{DFA}(k, 0)$$

Automata are

well-studied;

closely related to complexity;

a place where many fundamental techniques have been discovered;

implicit in an *unexpected* way;

explicit in an *expected* way.

The *quality* of their storage impacts directly on their expressive power.

Inspiration to formalize computation in GoI (Aubert and Seiller, 2014; Aubert and Seiller, 2015).

Theorem (Aubert, Bagnol, Pistone, and Seiller, 2014)

“Logic programs where all the variables are at the same height characterize Logarithmic space.”

Theorem (Aubert, Bagnol, and Seiller, 2015)

“Logic programs using only unary functions characterize polynomial time.”

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Theorem (Aubert, Bagnol, and Seiller, 2015)




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Thanks!






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