Reversible Barbed Congruence on Configuration Structures

Groupe de Travail Lacl

Clément Aubert[1,3] Ioana Cristescu[2,3]

1

2

[3]ANR-14-CE25-0005 ELICA & ANR-11-INSE-0007 REVER

22 mai 2015

$$P = (a.b.0) + (b.a.0)$$

$$Q = (a.0)|(b.0)$$

$$P = (a.b.0) + (b.a.0) \xrightarrow{a} b.0$$

$$Q = (a.0)|(b.0)$$

$$P = (a.b.0) + (b.a.0) \xrightarrow{a} b.0$$

$$P = (a.b.0) + (b.a.0) \xrightarrow{b} a.0$$

$$Q = (a.0)|(b.0)$$

$$P = (a.b.0) + (b.a.0) \quad \overset{a}{\nearrow}\; b.0 \qquad \underset{b}{\searrow}\; a.0$$

$$Q = (a.0)|(b.0) \xrightarrow{a} 0|(b.0)$$

$$P = (a.b.0) + (b.a.0) \begin{cases} \xrightarrow{a} b.0 \\ \xrightarrow{b} a.0 \end{cases}$$

$$Q = (a.0)|(b.0) \begin{cases} \xrightarrow{a} 0|(b.0) \\ \xrightarrow{b} (a.0)|0 \end{cases}$$

$$P = (a.b.0) + (b.a.0) \xrightarrow{a} b.0$$

$$P = (a.b.0) + (b.a.0) \xrightarrow[b]{} a.0$$

$$P|0 \equiv P$$

$$P|Q \equiv Q|P$$

$$P + 0 \equiv P$$

$$Q = (a.0)|(b.0) \xrightarrow{a} 0|(b.0)$$

$$Q = (a.0)|(b.0) \xrightarrow[b]{} (a.0)|0$$

$$P = (a.b.0) + (b.a.0) \xrightarrow{a} b.0$$

$$P = (a.b.0) + (b.a.0) \xrightarrow{b} a.0$$

$$P|0 \equiv P$$

$$P|Q \equiv Q|P$$

$$P + 0 \equiv P$$

$$Q = (a.0)|(b.0) \xrightarrow{a} b.0$$

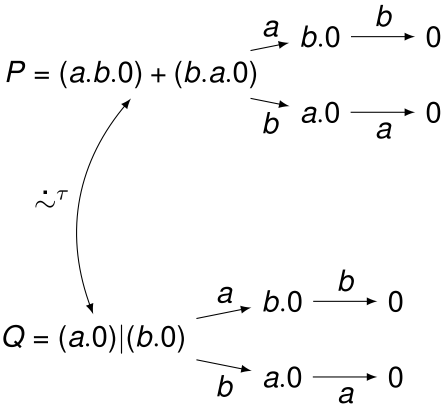
$$Q = (a.0)|(b.0) \xrightarrow{b} a.0$$

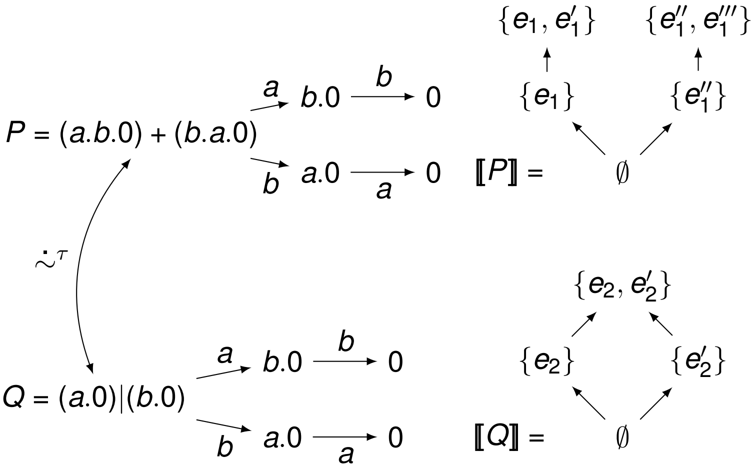
$$P = (a.b.0) + (b.a.0) \xrightarrow{a} b.0 \xrightarrow{b} 0$$

$$P = (a.b.0) + (b.a.0) \xrightarrow{b} a.0 \xrightarrow{a} 0$$

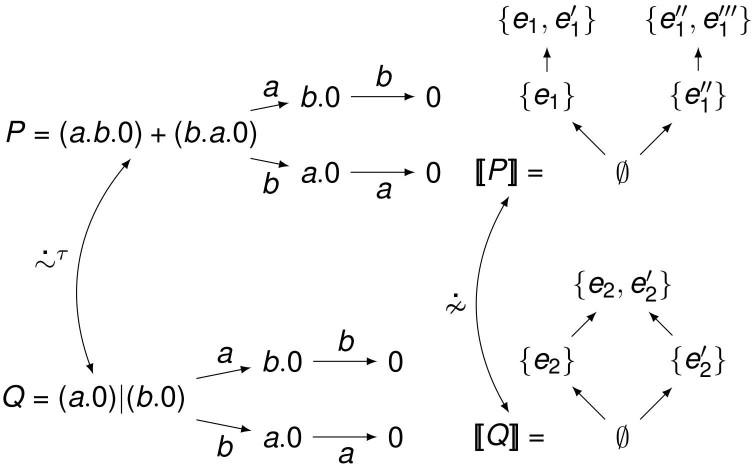
$$Q = (a.0)|(b.0) \xrightarrow{a} b.0 \xrightarrow{b} 0$$

$$Q = (a.0)|(b.0) \xrightarrow{b} a.0 \xrightarrow{a} 0$$

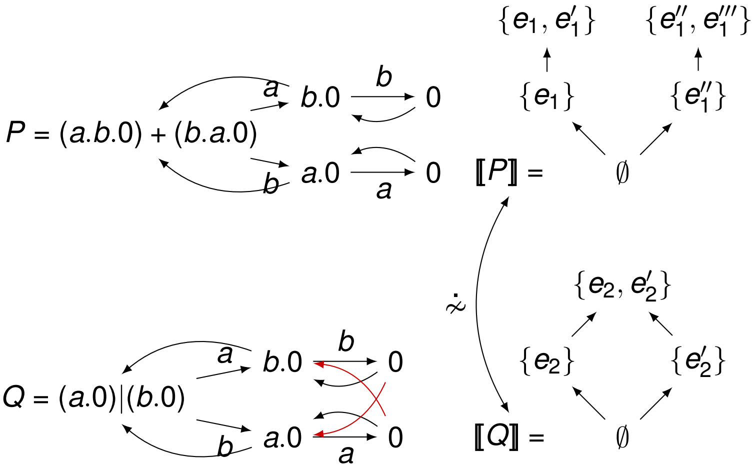

P = (a.b.0) + (b.a.0)
a
b.0
b
0
b
a.0
a
0
∼̇τ
Q = (a.0)|(b.0)
a
b.0
b
0
b
a.0
a
0

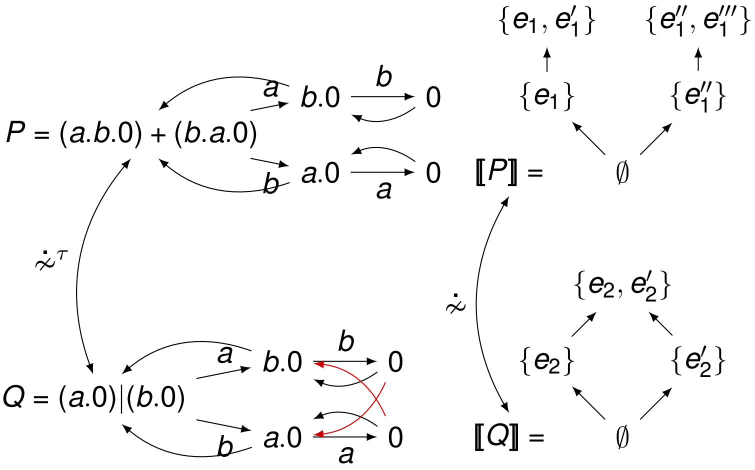


$P = (a.b.0) + (b.a.0)$
a
$b.0$
b
0
b
$a.0$
a
0
$[\![P]\!] =$
$\emptyset$
$\{e_1\}$
$\{e_1''\}$
$\{e_1, e_1'\}$
$\{e_1'', e_1'''\}$
$\dot{\sim}^{\tau}$
$Q = (a.0)|(b.0)$
a
$b.0$
b
0
b
$a.0$
a
0
$[\![Q]\!] =$
$\emptyset$
$\{e_2\}$
$\{e_2'\}$
$\{e_2, e_2'\}$



P = (a.b.0) + (b.a.0)
a
b.0
b
0
b
a.0
a
0
Q = (a.0)|(b.0)
a
b.0
b
0
b
a.0
a
0
⟦P⟧ =
∅
{e1}
{e1''}
{e1, e1'}
{e1'', e1'''}
⟦Q⟧ =
∅
{e2}
{e2'}
{e2, e2'}


P = (a.b.0) + (b.a.0)
a
b
b.0
a.0
0
0
[[P]] =
∅
{e1}
{e1''}
{e1, e1'}
{e1'', e1'''}
≁
Q = (a.0)|(b.0)
a
b
b.0
a.0
0
0
[[Q]] =
∅
{e2}
{e2'}
{e2, e2'}


$P = (a.b.0) + (b.a.0)$
a
$b.0$
b
0
b
$a.0$
a
0
$\dot{\approx}^{\tau}$
$Q = (a.0)|(b.0)$
a
$b.0$
b
0
b
$a.0$
a
0
$\{e_1, e_1'\}$
$\{e_1''\, e_1'''\}$
$\{e_1\}$
$\{e_1''\}$
$[\![P]\!] =$
$\emptyset$
$\dot{\approx}$
$\{e_2, e_2'\}$
$\{e_2\}$
$\{e_2'\}$
$[\![Q]\!] =$
$\emptyset$

Definition (Contextual Equivalence)

$$P \sim^{\tau} Q \Leftrightarrow \forall C[\cdot], C[P] \sim^{\tau} C[Q]$$

Question

Is Hereditary History-Preserving Bisimulation (hhpb) a contextual equivalence?

Definition (Contextual Equivalence)

$$P \sim^{\tau} Q \Leftrightarrow \forall C[\cdot], C[P] \sim^{\tau} C[Q]$$

Question

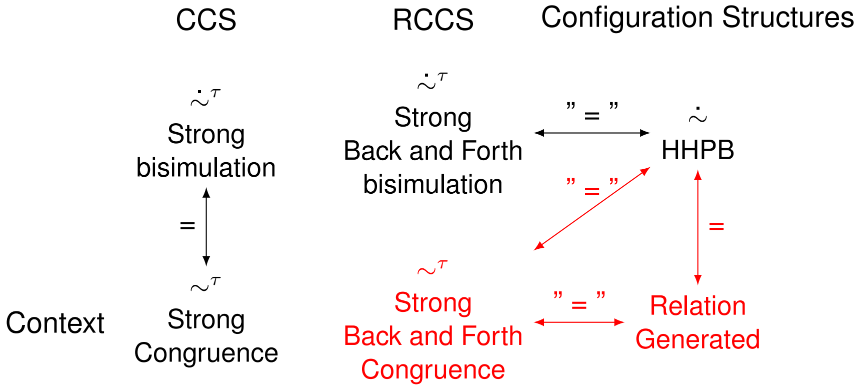
Is Hereditary History-Preserving Bisimulation (hhpb) a contextual equivalence?

Tools

- Context for RCCS and for configuration structures,
- Configuration structures for RCCS,
- (Co-)inductive approximation of relations.

Answer (modulo our tools)

Yes.


CCS
RCCS
Configuration Structures
$\dot{\sim}^{\tau}$
Strong bisimulation
=
$\sim^{\tau}$
Strong Congruence
Context
$\dot{\sim}^{\tau}$
Strong Back and Forth bisimulation
" = "
$\dot{\sim}$
HHPB
" = "
=
$\sim^{\tau}$
Strong Back and Forth Congruence
" = "
Relation Generated

$$\alpha, \beta := a \;\|\; \overline{a} \;\|\; \ldots \qquad \text{(Actions)}$$

$$P, Q := 0 \;\|\; a.P \;\|\; a.P + b.Q \;\|\; P|Q \qquad \text{(CCS processes)}$$

$$C := [\,] \;\|\; a.C \;\|\; C + P \;\|\; C|P \qquad \text{(Context)}$$

Calculus of Communicating Systems (CCS) in one slide

$$\alpha, \beta := a \;\|\; \overline{a} \;\|\; \ldots \qquad \text{(Actions)}$$

$$P, Q := 0 \;\|\; a.P \;\|\; a.P + b.Q \;\|\; P|Q \qquad \text{(CCS processes)}$$

$$C := [\,] \;\|\; a.C \;\|\; C + P \;\|\; C|P \qquad \text{(Context)}$$

$$\ldots \qquad \frac{}{\alpha.P \xrightarrow{\alpha} P} \qquad \frac{P \xrightarrow{\alpha} P' \qquad Q \xrightarrow{\overline{\alpha}} Q'}{P|Q \xrightarrow{\tau} P'|Q'} \text{ syn.} \qquad \ldots$$

$$\alpha, \beta := a \;\|\; \overline{a} \;\|\; \ldots \qquad \text{(Actions)}$$

$$P, Q := 0 \;\|\; a.P \;\|\; a.P + b.Q \;\|\; P|Q \qquad \text{(CCS processes)}$$

$$C := [\,] \;\|\; a.C \;\|\; C + P \;\|\; C|P \qquad \text{(Context)}$$

$$\ldots \qquad \frac{}{\alpha.P \xrightarrow{\alpha} P} \qquad \frac{P \xrightarrow{\alpha} P' \qquad Q \xrightarrow{\overline{\alpha}} Q'}{P|Q \xrightarrow{\tau} P'|Q'} \text{ syn.} \qquad \ldots$$

Definition (Strong commitment (barb))

$P \downarrow_\alpha$ if there exists P' such that $P \xrightarrow{\alpha} P'$.

$$\alpha, \beta := a \;\|\; \overline{a} \;\|\; \ldots \qquad \text{(Actions)}$$

$$P, Q := 0 \;\|\; a.P \;\|\; a.P + b.Q \;\|\; P|Q \qquad \text{(CCS processes)}$$

$$C := [\,] \;\|\; a.C \;\|\; C + P \;\|\; C|P \qquad \text{(Context)}$$

$$\ldots \qquad \frac{}{\alpha.P \xrightarrow{\alpha} P} \qquad \frac{P \xrightarrow{\alpha} P' \qquad Q \xrightarrow{\overline{\alpha}} Q'}{P|Q \xrightarrow{\tau} P'|Q'} \text{ syn.} \qquad \ldots$$

Definition (Strong barbed (bisimulation | congruence))

$$P \dot{\sim}^{\tau} Q \Leftrightarrow \begin{cases} Q \dot{\sim}^{\tau} P \\ P \xrightarrow{\tau} P' \Rightarrow Q \xrightarrow{\tau} Q' \wedge P' \dot{\sim}^{\tau} Q' \\ P \downarrow_{\alpha} \Rightarrow Q \downarrow_{\alpha} \end{cases}$$

$$\alpha, \beta := a \ \| \ \overline{a} \ \| \ \ldots \qquad \text{(Actions)}$$

$$P, Q := 0 \ \| \ a.P \ \| \ a.P + b.Q \ \| \ P|Q \qquad \text{(CCS processes)}$$

$$C := [\,] \ \| \ a.C \ \| \ C + P \ \| \ C|P \qquad \text{(Context)}$$

$$\ldots \qquad \frac{}{\alpha.P \xrightarrow{\alpha} P} \qquad \frac{P \xrightarrow{\alpha} P' \qquad Q \xrightarrow{\overline{\alpha}} Q'}{P|Q \xrightarrow{\tau} P'|Q'} \text{ syn.} \qquad \ldots$$

Definition (Strong barbed (bisimulation | congruence))

$$P \stackrel{\cdot}{\sim}^{\tau} Q \Leftrightarrow \begin{cases} Q \stackrel{\cdot}{\sim}^{\tau} P \\ P \xrightarrow{\tau} P' \Rightarrow Q \xrightarrow{\tau} Q' \wedge P' \stackrel{\cdot}{\sim}^{\tau} Q' \\ P \downarrow_{\alpha} \Rightarrow Q \downarrow_{\alpha} \end{cases}$$

$$P \sim^{\tau} Q \Leftrightarrow \forall C, C[P] \stackrel{\cdot}{\sim}^{\tau} C[Q]$$

$$R, S := m \triangleright P \;\parallel\; R|R \qquad \text{(RCCS processes)}$$

$$m := \emptyset \;\parallel\; \curlyvee .m \;\parallel\; \langle i, a, P\rangle.m \;\parallel\; \langle i, a\rangle.m \qquad \text{(Memories)}$$

$$R, S := m \triangleright P \;\|\; R|R \qquad \text{(RCCS processes)}$$

$$m := \emptyset \;\|\; \curlyvee .m \;\|\; \langle i, a, P\rangle.m \;\|\; \langle i, a\rangle.m \qquad \text{(Memories)}$$

Example

$$\emptyset \triangleright (a.P + b.Q)|(c.\overline{a}.P')$$

$$R, S := m \rhd P \parallel R|R \qquad \text{(RCCS processes)}$$
$$m := \emptyset \parallel \curlyvee.m \parallel \langle i, a, P\rangle.m \parallel \langle i, a\rangle.m \qquad \text{(Memories)}$$

Example

$$\emptyset \rhd (a.P + b.Q)|(c.\overline{a}.P')$$
$$\equiv (\curlyvee.\emptyset \rhd (a.P + b.Q))|(\curlyvee.\emptyset \rhd (c.\overline{a}.P'))$$

$$R, S := m \triangleright P \;\|\; R|R \qquad \text{(RCCS processes)}$$

$$m := \emptyset \;\|\; \curlyvee.m \;\|\; \langle i, a, P\rangle.m \;\|\; \langle i, a\rangle.m \qquad \text{(Memories)}$$

Example

$$\emptyset \triangleright (a.P + b.Q)|(c.\overline{a}.P')$$
$$\equiv (\curlyvee.\emptyset \triangleright (a.P + b.Q))|(\curlyvee.\emptyset \triangleright (c.\overline{a}.P'))$$
$$\rightarrow^{1:b} (\langle 1, b, a.P\rangle.\curlyvee.\emptyset \triangleright Q)|(\curlyvee.\emptyset \triangleright (c.\overline{a}.P'))$$

$$R, S := m \rhd P \;\|\; R|R \qquad \text{(RCCS processes)}$$
$$m := \emptyset \;\|\; \curlyvee.m \;\|\; \langle i, a, P\rangle.m \;\|\; \langle i, a\rangle.m \qquad \text{(Memories)}$$

Example

$$\emptyset \rhd (a.P + b.Q)|(c.\overline{a}.P')$$
$$\equiv (\curlyvee.\emptyset \rhd (a.P + b.Q))|(\curlyvee.\emptyset \rhd (c.\overline{a}.P'))$$
$$\rightarrow^{1:b}(\langle 1, b, a.P\rangle.\curlyvee.\emptyset \rhd Q)|(\curlyvee.\emptyset \rhd (c.\overline{a}.P'))$$
$$\rightarrow^{2:c}(\langle 1, b, a.P\rangle.\curlyvee.\emptyset \rhd Q)|(\langle 2, c\rangle.\curlyvee.\emptyset \rhd (\overline{a}.P'))$$

$$R, S := m \triangleright P \;\|\; R|R \qquad \text{(RCCS processes)}$$

$$m := \emptyset \;\|\; \curlyvee.m \;\|\; \langle i, a, P\rangle.m \;\|\; \langle i, a\rangle.m \qquad \text{(Memories)}$$

Example

$$\emptyset \triangleright (a.P + b.Q)|(c.\overline{a}.P')$$
$$\equiv (\curlyvee.\emptyset \triangleright (a.P + b.Q))|(\curlyvee.\emptyset \triangleright (c.\overline{a}.P'))$$
$$\xrightarrow{1:b} (\langle 1, b, a.P\rangle.\curlyvee.\emptyset \triangleright Q)|(\curlyvee.\emptyset \triangleright (c.\overline{a}.P'))$$
$$\xrightarrow{2:c} (\langle 1, b, a.P\rangle.\curlyvee.\emptyset \triangleright Q)|(\langle 2, c\rangle.\curlyvee.\emptyset \triangleright (\overline{a}.P'))$$
$$\rightsquigarrow^{1:b} (\curlyvee.\emptyset \triangleright (a.P + b.Q))|(\langle 2, c\rangle.\curlyvee.\emptyset \triangleright (\overline{a}.P'))$$

$$R, S := m \triangleright P \ \parallel \ R|R \qquad \text{(RCCS processes)}$$

$$m := \emptyset \ \parallel \ \curlyvee.m \ \parallel \ \langle i, a, P\rangle.m \ \parallel \ \langle i, a\rangle.m \qquad \text{(Memories)}$$

Example

$$\emptyset \triangleright (a.P + b.Q)|(c.\overline{a}.P')$$

$$\equiv (\curlyvee.\emptyset \triangleright (a.P + b.Q))|(\curlyvee.\emptyset \triangleright (c.\overline{a}.P'))$$

$$\rightarrow^{1:b}(\langle 1, b, a.P\rangle.\curlyvee.\emptyset \triangleright Q)|(\curlyvee.\emptyset \triangleright (c.\overline{a}.P'))$$

$$\rightarrow^{2:c}(\langle 1, b, a.P\rangle.\curlyvee.\emptyset \triangleright Q)|(\langle 2, c\rangle.\curlyvee.\emptyset \triangleright (\overline{a}.P'))$$

$$\rightsquigarrow^{1:b}(\curlyvee.\emptyset \triangleright (a.P + b.Q))|(\langle 2, c\rangle.\curlyvee.\emptyset \triangleright (\overline{a}.P'))$$

$$\rightarrow^{3:\tau}(\langle 3, a, b.Q\rangle.\curlyvee.\emptyset \triangleright P)|(\langle 3, \overline{a}\rangle.\langle 2, c\rangle.\curlyvee.\emptyset \triangleright P')$$

$$R, S := m \rhd P \parallel R | R \qquad \text{(RCCS processes)}$$

$$m := \emptyset \parallel \curlyvee .m \parallel \langle i, a, P \rangle .m \parallel \langle i, a \rangle .m \qquad \text{(Memories)}$$

Definition (Strong back-and-forth barbed bisimulation)

$$R \dot{\sim}^{\tau} S \Leftrightarrow \begin{cases} S \dot{\sim}^{\tau} R \\ R \twoheadrightarrow^{\tau} R' \Rightarrow S \twoheadrightarrow^{\tau} S' \wedge R' \dot{\sim}^{\tau} S' \\ R \rightsquigarrow^{\tau} R' \Rightarrow S \rightsquigarrow^{\tau} S' \wedge R' \dot{\sim}^{\tau} S' \\ R \downarrow_a \Rightarrow S \downarrow_a \end{cases}$$

$$R, S := m \triangleright P \;\parallel\; R|R \qquad \text{(RCCS processes)}$$

$$m := \emptyset \;\parallel\; \curlyvee.m \;\parallel\; \langle i, a, P\rangle.m \;\parallel\; \langle i, a\rangle.m \qquad \text{(Memories)}$$

Definition (Strong back-and-forth barbed bisimulation)

$$R \mathrel{\dot{\sim}^\tau} S \Leftrightarrow \begin{cases} S \mathrel{\dot{\sim}^\tau} R \\ R \xrightarrow{}{}^{\tau} R' \Rightarrow S \xrightarrow{}{}^{\tau} S' \wedge R' \mathrel{\dot{\sim}^\tau} S' \\ R \rightsquigarrow^{\tau} R' \Rightarrow S \rightsquigarrow^{\tau} S' \wedge R' \mathrel{\dot{\sim}^\tau} S' \\ R \downarrow_a \Rightarrow S \downarrow_a \end{cases}$$

What about congruence?

Definition (Origin of a process)

$$O_R = \emptyset \rhd P \twoheadrightarrow^* R$$

Simplest case: contexts and processes with an empty memory.

Definition (Origin of a process)

$$O_R = \emptyset \rhd P \rightarrow^* R$$

Simplest case: contexts and processes with an empty memory.

Definition (Strong back-and-forth barbed congruence)

$$R \sim^{\tau} S \Leftrightarrow \begin{cases} R \;\dot{\sim}^{\tau}\; S \\ \forall C[\cdot], C[O_R] \;\dot{\sim}^{\tau}\; C[O_S] \end{cases}$$

Definition (Origin of a process)

$$O_R = \emptyset \triangleright P \longrightarrow^* R$$

Simplest case: contexts and processes with an empty memory.

Definition (Strong back-and-forth barbed congruence)

$$R \sim^\tau S \Leftrightarrow \begin{cases} R \,\dot{\sim}^\tau\, S \\ \forall C[\cdot], C[O_R] \,\dot{\sim}^\tau\, C[O_S] \end{cases}$$

Example

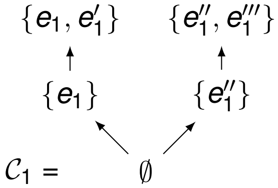
$$R = \langle i, b, a.P\rangle.\emptyset \triangleright Q \qquad S = \langle j, a, b.Q\rangle.\emptyset \triangleright P$$

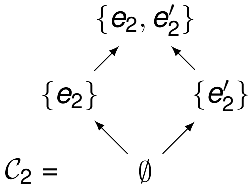
$O_R = O_S = \emptyset \triangleright a.P + b.Q$, so $O_R \sim^\tau O_S$ but $R \,\dot{\not\sim}^\tau\, S$!

Definition (Labelled configuration structure)

$\mathcal{C} = \langle E, C, \ell \rangle$ with $C \subset \mathcal{P}(E)$ and $\ell : E \rightarrow$ labels.

Example





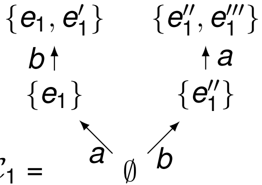
$\ell_1(e_1) = \ell_1(e_1''') = a$
$\ell_1(e_1') = \ell_1(e_1'') = b$

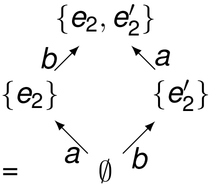
$\ell_2(e_2) = a$
$\ell_2(e_2') = b$

Definition (Labelled configuration structure)

$\mathcal{C} = \langle E, C, \ell \rangle$ with $C \subset \mathcal{P}(E)$ and $\ell : E \rightarrow$ labels.

Example

$\mathcal{C}_1 =$ 

$\mathcal{C}_2 =$ 

$\ell_1(e_1) = \ell_1(e_1''') = a$

$\ell_1(e_1') = \ell_1(e_1'') = b$

$\ell_2(e_2) = a$

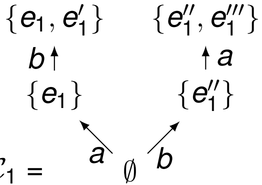
$\ell_2(e_2') = b$

Definition (Labelled configuration structure)

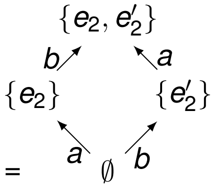
$\mathcal{C} = \langle E, C, \ell \rangle$ with $C \subset \mathcal{P}(E)$ and $\ell : E \to$ labels.

Example

$\mathcal{C}_1 =$



$\mathcal{C}_2 =$



$\emptyset, \emptyset, \emptyset$

$\{e_1\}, \{e_2\}, e_1 \leftrightarrow e_2$

$\{e_1, e'_1\}, \{e_2, e'_2\}, e'_1 \leftrightarrow e'_2$

$\{e''_1\}, \{e'_2\}, e''_1 \leftrightarrow ?$

Definition (Labelled configuration structure)

$\mathcal{C} = \langle E, C, \ell \rangle$ with $C \subset \mathcal{P}(E)$ and $\ell : E \rightarrow$ labels.

Definition (HHPB)

$(\emptyset, \emptyset, \emptyset) \in \mathcal{R}$, and for $x_i \in C_i, e_i \in E_i, (x_1, x_2, f) \in \mathcal{R} \Rightarrow$

$$\begin{cases} f \text{ label and order preserving bijection} \\ \\ \end{cases}$$

Definition (Labelled configuration structure)

$\mathcal{C} = \langle E, C, \ell \rangle$ with $C \subset \mathcal{P}(E)$ and $\ell : E \to$ labels.

Definition (HHPB)

$(\emptyset, \emptyset, \emptyset) \in \mathcal{R}$, and for $x_i \in C_i, e_i \in E_i, (x_1, x_2, f) \in \mathcal{R} \Rightarrow$

$$\begin{cases} f \text{ label and order preserving bijection} \\ x_1 \xrightarrow{\alpha} x_1 \cup \{e_1\} \implies x_2 \xrightarrow{\alpha} x_2 \cup \{e_2\}, \\ \end{cases}$$

Definition (Labelled configuration structure)

$\mathcal{C} = \langle E, C, \ell \rangle$ with $C \subset \mathcal{P}(E)$ and $\ell : E \rightarrow$ labels.

Definition (HHPB)

$(\emptyset, \emptyset, \emptyset) \in \mathcal{R}$, and for $x_i \in C_i, e_i \in E_i, (x_1, x_2, f) \in \mathcal{R} \Rightarrow$

$$\begin{cases} f \text{ label and order preserving bijection} \\ x_1 \xrightarrow{\alpha} x_1 \cup \{e_1\} \implies x_2 \xrightarrow{\alpha} x_2 \cup \{e_2\}, (x_1 \cup \{e_1\}, x_2 \cup \{e_1\}, f') \in \mathcal{R} \\ \end{cases}$$

Definition (Labelled configuration structure)

$\mathcal{C} = \langle E, C, \ell \rangle$ with $C \subset \mathcal{P}(E)$ and $\ell : E \rightarrow$ labels.

Definition (HHPB)

$(\emptyset, \emptyset, \emptyset) \in \mathcal{R}$, and for $x_i \in C_i, e_i \in E_i, (x_1, x_2, f) \in \mathcal{R} \Rightarrow$

$$\begin{cases} f \text{ label and order preserving bijection} \\ x_1 \rightarrow^{\alpha} x_1 \cup \{e_1\} \implies x_2 \rightarrow^{\alpha} x_2 \cup \{e_2\}, (x_1 \cup \{e_1\}, x_2 \cup \{e_1\}, f') \in \mathcal{R} \\ x_1 \rightsquigarrow^{\alpha} x_1 \setminus \{e_1\} \implies x_2 \rightsquigarrow^{\alpha} x_2 \setminus \{e_2\}, (x_1 \setminus \{e_1\}, x_2 \setminus \{e_1\}, f') \in \mathcal{R} \end{cases}$$

Reversible Configuration Structures

$$O_R = \emptyset \rhd \underbrace{P \xrightarrow{\alpha_1} \dots \xrightarrow{\alpha_i}}_{x_R=\{e_1,\dots,e_i\}} R \qquad [\![R]\!] = ([\![O_R]\!], x_R)$$

We can consider only forward transitions.

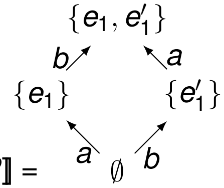
Reversible Configuration Structures

$$O_R = \emptyset \rhd \underbrace{P \xrightarrow{\alpha_1} \ldots \xrightarrow{\alpha_i}}_{x_R=\{e_1,\ldots,e_i\}} R \qquad [\![R]\!] = ([\![O_R]\!], x_R)$$

We can consider only forward transitions.

Example

$[\![P]\!] =$



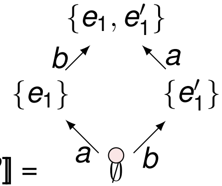
$P = \quad a.0 | \quad b.0$

Reversible Configuration Structures

$$O_R = \emptyset \triangleright P \underbrace{\xrightarrow{\alpha_1} \ldots \xrightarrow{\alpha_i}}_{x_R=\{e_1,\ldots,e_i\}} R \qquad [\![R]\!] = ([\![O_R]\!], x_R)$$

We can consider only forward transitions.

Example

$[\![R]\!] =$ 

$$\emptyset \triangleright P \equiv \curlyvee .\emptyset \triangleright a.0 |\ \curlyvee .\emptyset \triangleright b.0$$

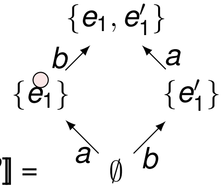
Reversible Configuration Structures

$$O_R = \emptyset \triangleright P \underbrace{\xrightarrow{\alpha_1} \dots \xrightarrow{\alpha_i}}_{x_R=\{e_1,\dots,e_i\}} R \qquad [\![R]\!] = ([\![O_R]\!], x_R)$$

We can consider only forward transitions.

Example

$[\![R]\!] =$



$$\emptyset \triangleright P \equiv \Upsilon.\emptyset \triangleright a.0 \mid \Upsilon.\emptyset \triangleright b.0$$

$$\xrightarrow{1:a} \langle 1, a \rangle.\Upsilon.\emptyset \triangleright 0 \mid \Upsilon.\emptyset \triangleright b.0$$

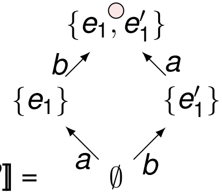
Reversible Configuration Structures

$$O_R = \emptyset \triangleright P \underbrace{\xrightarrow{\alpha_1} \dots \xrightarrow{\alpha_i}}_{x_R = \{e_1, \dots, e_i\}} R \qquad [\![R]\!] = ([\![O_R]\!], x_R)$$

We can consider only forward transitions.

Example

$[\![R]\!] =$



$$\emptyset \triangleright P \equiv \curlyvee.\emptyset \triangleright a.0 | \curlyvee.\emptyset \triangleright b.0$$
$$\xrightarrow{1:a} \langle 1, a \rangle . \curlyvee.\emptyset \triangleright 0 | \curlyvee.\emptyset \triangleright b.0$$
$$\xrightarrow{2:b} \langle 1, a \rangle . \curlyvee.\emptyset \triangleright 0 | \langle 2, b \rangle . \curlyvee.\emptyset \triangleright 0$$

Reversible Configuration Structures

$$O_R = \emptyset \triangleright P \underbrace{\xrightarrow{\alpha_1} \ldots \xrightarrow{\alpha_i}}_{x_R=\{e_1,\ldots,e_i\}} R \qquad [\![R]\!] = ([\![O_R]\!], x_R)$$

We can consider only forward transitions.

Lemma (Operational correspondence)

For $\twoheadrightarrow \in \{\rightarrow, \rightsquigarrow\}$, *let* $[\![R]\!] = (\mathcal{C}, x)$, $\gamma \in \{\alpha, \tau\}$:

$$R \twoheadrightarrow^{i:\gamma} S \Rightarrow [\![R]\!] \twoheadrightarrow^{\gamma} [\![S]\!]$$

$$[\![R]\!] \twoheadrightarrow^{\gamma} (\mathcal{C}, x \cup \{e\}) \Rightarrow R \twoheadrightarrow^{i:\gamma} S \text{ and } [\![S]\!] = (\mathcal{C}, x \cup \{e\}).$$

RCCS

Configuration Structures

$R \sim^{\tau} S$

$([\![O_R]\!], x_R)\text{''} \sim \text{''}([\![O_S]\!], x_S)$

$\Downarrow$

$O_R \sim^{\tau} O_S \Longleftrightarrow [\![O_R]\!] \sim^{\tau} [\![O_S]\!] \Longleftrightarrow [\![O_R]\!] \dot{\sim} [\![O_S]\!]$

Context with memory?
Weak?
Restriction?
What to observe? Directions?